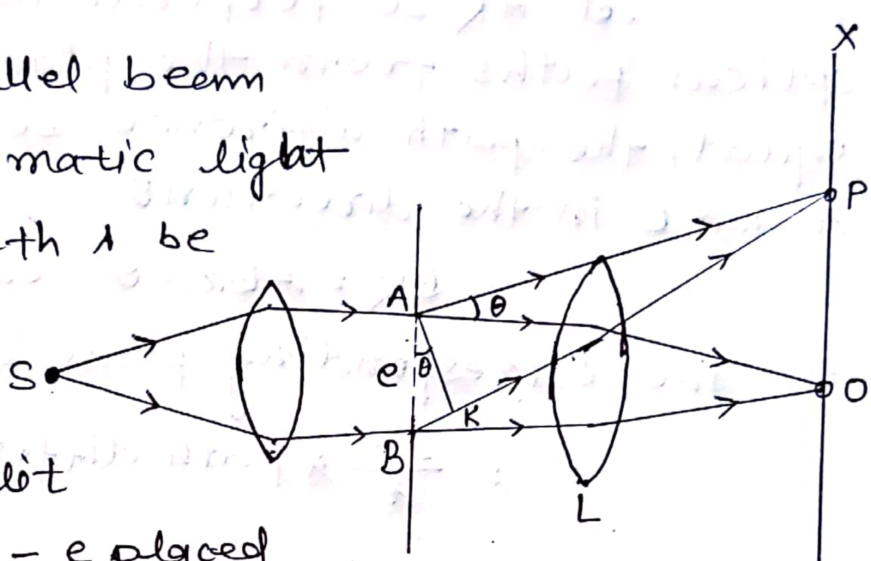
prof. Vidya Sagar
Department of physics
B. Sc(part-II) Hons

Fraunhofer's diffraction due to a Single Slit of width 'e'.



Let a parallel beam of monochromatic light of wavelength λ be incident Normally upon a narrow slit of width AB = e placed perpendicular to the plane of paper. Let the diffracted light be focussed by a ~~Concave~~ Convex lens L on a Screen XY placed in the focal plane of the lens. The diffraction pattern obtained on the Screen Consists of a Central bright and band, having a alternate dark and weak bright band of decreasing intensity on both sides.

Explanation :- In terms of wave theory, a plane wave front is ~~a~~ incident normally on the slit AB. According to Huygen's principal, each point in AB sends out secondary wavelets in all direction. The ray proceeding in the same direction as the incident rays are focussed at O. while those diffracted through an angle θ are focussed at P.

Let AK be perpendicular to BK. As the optical paths from the plane AK to P are equal, the path diffrence between the wavelets A and B in the direction θ.

$$BK = AB \sin\theta = e\sin\theta$$

The corresponding phase difference is

$$= \frac{2\pi}{\lambda} \times \text{path difference}$$

$$= \frac{2\pi}{\lambda} \times (e\sin\theta)$$

Let the width AB of the slit divided in to n equal parts. The amplitude of vibration at P due to the waves from each part will be the same, say equal to a. The phase diffrence between the waves from any two consecutive part is $\frac{1}{n}\left(\frac{2\pi}{\lambda} e\sin\theta\right) = d$ (let)

Hence the resultant amplitude at P is give by

$$R = \frac{a \sin(nd/2)}{\sin(d/2)}$$

$$R = \frac{a \sin\left(\frac{\pi e \sin\theta}{\lambda}\right)}{\sin\left(\frac{\pi e \sin\theta}{n\lambda}\right)}$$

Let us put $\frac{\pi e \sin\theta}{\lambda} = \alpha$

then $$R = \frac{a \sin\alpha}{\sin \alpha/n}$$

$$\therefore R = \frac{a\sin\alpha}{\alpha/n} \quad (\because \alpha/n \text{ is very small})$$

$$\therefore R = \frac{na \sin\alpha}{\alpha}$$

As $n \to \infty$; $a \to 0$ but the product na remains finite. Let $na = A$

Then
$$R = \frac{A \sin\alpha}{\alpha}$$

Therefore, the resultant intensity at p. being proportional ~~to~~ to the square of the amplitude, is

$$I = R^2$$

$$I = A^2 \left(\frac{\sin\alpha}{\alpha}\right)^2 \quad \text{——— (1)}$$

the Constant of proportionality being taken as unity for simplicity.

Direction of Maxima and Minima :— It is clear from eqn (1) that the intensity is a minimum when

$$\frac{\sin\alpha}{\alpha} = 0$$

or, $\sin\alpha = 0$

$$\therefore \alpha = \pm m\pi \quad (\text{where } m = 1, 2, 3, \ldots)$$

But $$\alpha = \frac{\pi e \sin\theta}{\lambda}$$

$$\therefore \quad \frac{\pi e \sin\theta}{\lambda} = \pm m\pi$$

or, $e\sin\theta = \pm m\lambda$ —— (2)

This equation gives the direction of the first, second, third, ---- minima by putting $m = 1, 2, 3, ---$

To find the direction of maximum intensity, let us, differentiate eqn (1) with respect to α and equate to zero i.e.

$$\frac{dI}{d\alpha} = 0$$

or, $$\frac{d}{d\alpha}\left[A^2\left(\frac{\sin\alpha}{\alpha}\right)^2\right] = 0$$

or, $$A^2\left(\frac{2\sin\alpha}{\alpha}\right)\frac{\alpha\cos\alpha - \sin\alpha}{\alpha^2} = 0$$

or, $$\frac{\alpha\cos\alpha - \sin\alpha}{\alpha^2} = 0$$

or, $$\alpha\cos\alpha - \sin\alpha = 0$$

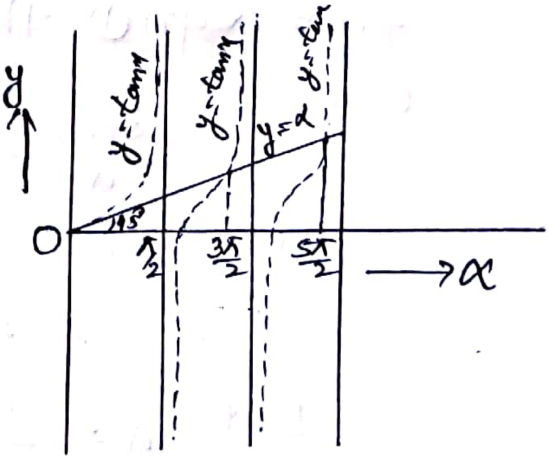
or, $$\alpha\cos\alpha = \sin\alpha$$

$$\alpha = \frac{\sin\alpha}{\cos\alpha}$$

$$\therefore \alpha = \tan\alpha$$

This equation is solved graphically by plotting the curves

$$y = \alpha$$
$$y = \tan\alpha \quad \text{—— (3)}$$



The first of these is a straight line through origin making an angle of 45°, while the second is a discontinuous curve having a number of branches. The points of intersection of the two gives the value of α Satisfying eqn ③. These values are approximately given out as.

$$\alpha = 0, \frac{3\pi}{2}, \frac{5\pi}{2}, \frac{7\pi}{2}, \text{------}$$

as more exactly as

$$\alpha = 0, 1.430\pi, 2.462\pi, 3.471\pi, \text{------}$$

Substituting the approximate values of α in eqn① we get the intensities of various maxima. Thus the intensity of the central maximum is

$$I_0 = A^2\left(\frac{\sin 0}{0}\right)^2 = A^2$$

Similarly, the intensity of the first subsidiary maxima is

$$I_1 \simeq A^2\left(\frac{\sin\frac{3\pi}{2}}{\frac{3\pi}{2}}\right)^2 = \frac{4}{9\pi^2}A^2 \simeq \frac{A^2}{22}$$

that of the second subsidiary maximum is

$$I_2 = A^2\left(\frac{\sin\frac{5\pi}{2}}{\frac{5\pi}{2}}\right)^2 \simeq \frac{4}{25\pi^2}A^2 \simeq \frac{A^2}{61}$$

and so on. Thus the intensities of successive are in the ratio

$$1 : \frac{4}{9\pi^2} : \frac{4}{25\pi^2} : \frac{4}{49\pi^2} : \text{--------}$$

$$\text{or, } 1 : \frac{1}{22} : \frac{1}{61} : \frac{1}{121} : \text{------}$$

clearly most of the incident light is concentrated in the principal maximum which occurs in the direction given by

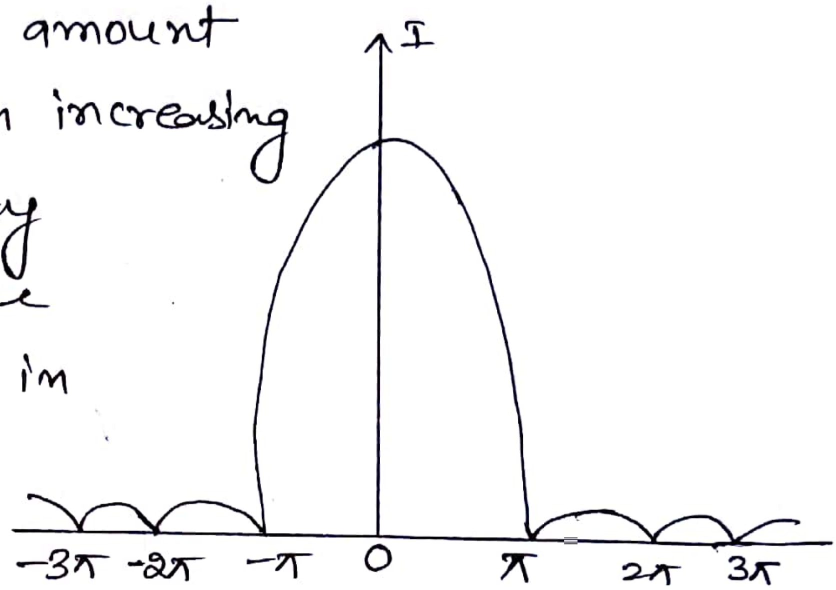
$$\alpha = 0$$

$$\text{or, } \frac{\pi e \sin\theta}{\lambda} = 0$$

or, $\theta = 0$ that is the same direction as the incident light.

Thus the diffraction pattern consists of a bright principal maximum in the direction of the incident light, having alternately minima and weak subsidiary maxima of rapidly decreasing intensity on either side of it. The minima lie at

$\alpha = \pm\pi, \pm 2\pi, \pm 3\pi, \ldots\ldots$. The subsidiary maxima do not fall exactly mid-way between minima but are displaced towards the centre of the pattern by an amount which decreases with increasing order. The intensity distribution in the pattern is shown in the figure.



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